$$m\vec{a} = \vec{W}_g \;\Rightarrow\; \begin{array}{l} mv_x \dfrac{dv_x}{dx} = -kv_x^2 \\ mv_y \dfrac{dv_y}{dy} = mg \end{array}$$

$$mv_x \frac{dv_x}{dx} = -kv_x^2 \;\Rightarrow\; -\frac{m}{k}\frac{dv_x}{v_x} = dx \;\Rightarrow\; \frac{dv_x}{v_x} = -\frac{k}{m}dx \,/ \int(\;) \;\Rightarrow\; \ln|v_x| = C_1 - \frac{k}{m}x$$

$$v_x(x, C_2) = C_2 e^{-\frac{k}{m}x} \quad v_x(0, C_2) = C_2 = v_0\cos\alpha \;\Rightarrow\; \boxed{v_x(x) = v_0\cos\alpha e^{-\frac{k}{m}x}} \,,\; \frac{dx}{dt} = v_0\cos\alpha e^{-\frac{k}{m}x} \;\Rightarrow\; e^{\frac{k}{m}x}dx = v_0\cos\alpha dt \,/ \int(\;)$$

$$\frac{m}{k}e^{\frac{k}{m}x} = v_0\cos\alpha t + C_3 \,,\; \frac{m}{k} = C_3 \;\Rightarrow\; \boxed{t(x) = \frac{m}{kv_0\cos\alpha}\left(e^{\frac{k}{m}x} - 1\right)}$$

$$v_y \frac{dv_y}{dy} = g \;\Rightarrow\; v_y dv_y = gdy \,/ \int(\;) \;\Rightarrow\; \frac{1}{2}v_y^2 = gy + C_4 \;\Rightarrow\; v_y(y, C_5) = \sqrt{2gy + C_5}$$

$$v_y(0, C_5) = v_0\sin\alpha = \sqrt{C_5} \;\Rightarrow\; \boxed{v_y(y) = \sqrt{2gy + v_0^2\sin^2\alpha} = \frac{dy}{dt}} \;\Rightarrow\; \frac{dy}{\sqrt{2gy + v_0^2\sin^2\alpha}} = dt \,/ \int(\;)$$

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$$\int \frac{1}{\sqrt{2gy + v_0^2\sin^2\alpha}}dy = \begin{pmatrix} 2gy + v_0^2\sin^2\alpha = \xi \\ 2gdy = d\xi \end{pmatrix} = \frac{1}{2g}\int \xi^{-\frac{1}{2}}d\xi = \frac{1}{g}\sqrt{2gy + v_0^2\sin^2\alpha} + C_6$$

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$$\Rightarrow\; \frac{1}{g}\sqrt{2gy + v_0^2\sin^2\alpha} + C_6 = t \,,\; y(0) = h \;\Rightarrow\; \frac{1}{g}\sqrt{2gh + v_0^2\sin^2\alpha} + C_6 = 0 \;\Rightarrow\; C_6 = -\frac{1}{g}\sqrt{2gh + v_0^2\sin^2\alpha}$$

$$\Rightarrow\; y(t) = \frac{1}{2g}\left[\left(gt + \sqrt{2gh + v_0^2\sin^2\alpha}\right)^2 - v_0^2\sin^2\alpha\right] \Rightarrow \boxed{y(x) = \frac{1}{2g}\left[\left(\frac{mg}{kv_0\cos\alpha}\left(e^{\frac{k}{m}x} - 1\right) + \sqrt{2gh + v_0^2\sin^2\alpha}\right)^2 - v_0^2\sin^2\alpha\right]}$$

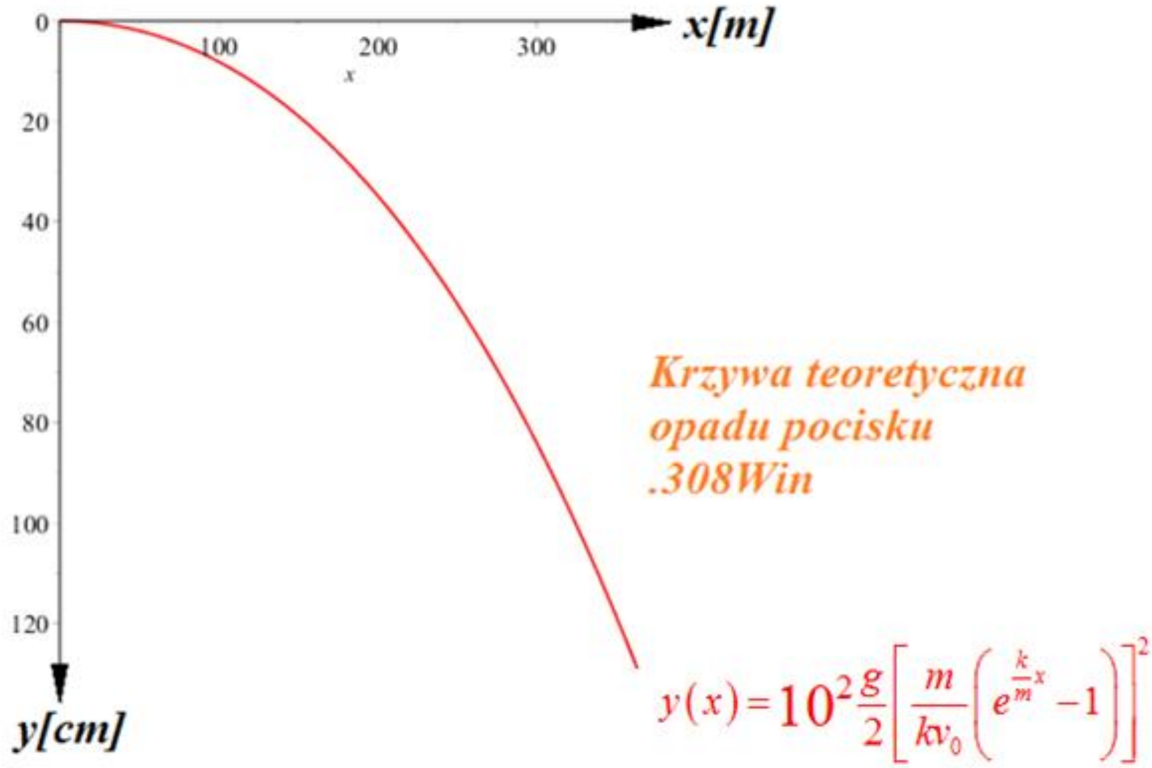
$$\text{strzal w poziomie :}\; \begin{array}{c} \alpha = 0 \\ (h = 0) \end{array} \;\Rightarrow\; \boxed{y(x) = 10^2 \cdot \frac{g}{2}\left[\frac{m}{kv_0}\left(e^{\frac{k}{m}x} - 1\right)\right]^2} [\mathrm{cm}]$$

$$k = \frac{1}{2} \cdot C_x \cdot \rho \cdot A = 0.5 \cdot 0.09 \cdot 1.2 \cdot \pi \cdot \frac{0.00762}{2} \cdot 0.0279 = 1.8 \cdot 10^{-5}\left[\frac{\mathrm{kg}}{\mathrm{m}}\right]$$

$$v_0 = 804\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

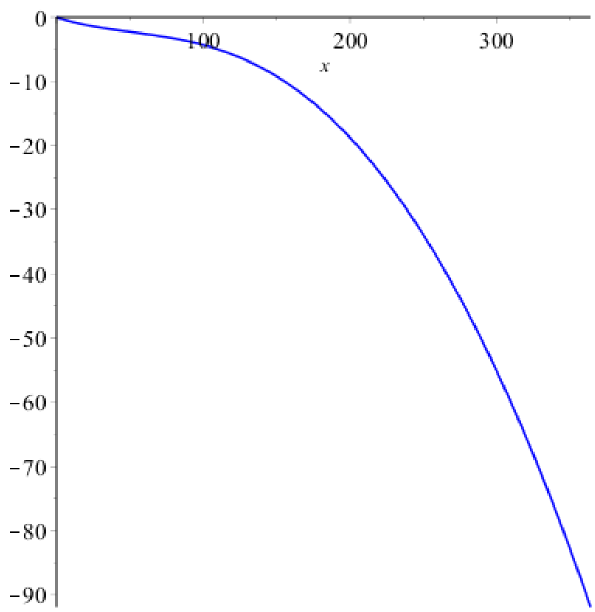
$$g = 9.81\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$$

$$m = 0.01089[\mathrm{kg}]$$

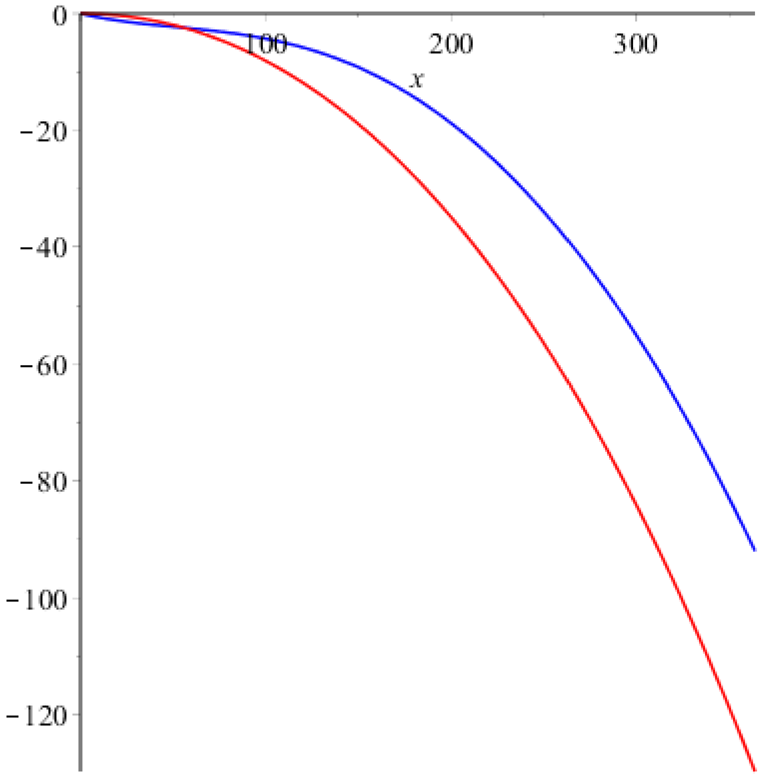


Interpolacja danych z pudełka amunicji "GGG":

$$y_{inetrpolacja}(x)=1.0576\cdot10^{-11}\cdot x^5-1.3653\cdot10^{-8}x^4+7.5142\cdot10^{-6}x^3-9.5021\cdot10^{-4}x^2+0.07568x$$



Teoria vs dane rzeczywiste :



aproksymacja : $\boxed{y(x) = \frac{2\pi}{13} \cdot 10^2 \, \frac{g}{2} \left[\frac{m}{k v_0} \left(e^{\frac{k}{m} x} - 1 \right) \right]^2}$ [cm]